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DESIGN AND FABRICATION OF AN ENGINEERING
MODEL FIBER-OPTICS DETECTOR

by

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ABSTRACT

The purpose of the work described herein has been to design and fabricate an annular ring detector consisting of optical fibers terminated with photodetectors. The maximum width of each concentric ring was to be small enough to permit the resolution of a Ronchi ruling transform with a dot spacing of 150 μm (0.0059 inch)^{*}. There were to be a minimum of 100 concentric rings covering a circular area of 2.54 cm (1.00 inch) diameter.

A fiber-optic array consisting of approximately 89,000 fibers of 76 μm (0.003 inch) diameter was fabricated to meet the above requirements. The fibers within a circular area of 2.56 cm (1.008 inches) diameter were sorted into 168 adjacent rings concentric with the center fiber.

The response characteristics of several photodetectors were measured, and the data used to compare their linearity of response and dynamic range. Also coupling loss measurements were made for three different methods of terminating the optical fibers with a photodetector.

To facilitate high speed scanning, an electronic multiplexing circuit which would allow the 169 data channels to be scanned at a 30 Hz rate was also designed. It was mutually agreed that NASA would implement the multiplexing circuit and terminate the optical fibers with photodiodes.

*The English system of units was used for the principal measurements and calculations.

A potential application of the annular ring detector was investigated by performing preliminary tests on a matrix technique for inverting measurements of light incident on the annular rings to data characteristic of objects in a configuration suitable for optical processing.

The axially symmetric array of optical-fibers and the computer programs used to test the matrix inversion technique were delivered to NASA in December, 1972.

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DESIGN AND FABRICATION OF AN ENGINEERING
MODEL FIBER-OPTICS DETECTOR

I. Introduction

Prior to beginning the development program described herein, work had been done at Georgia Tech using a comparatively simple fiber-optics array for measuring radial distributions of light intensity for particle size analysis [1,2,3]. This technique makes use of a 632.8-nm helium-neon laser beam and a lens to produce Fraunhofer diffraction patterns from particle suspensions. Because the suspensions of interest consist of large numbers of particles randomly located and randomly oriented, an axially symmetric distribution of light intensity in the diffraction pattern was assumed even for nonspherical particles. Thus the intensity of scattered light in the far-field was assumed to be a function of polar scattering angle only.

This symmetry determined the choice of a design geometry for the instrument.

In order to obtain an average of the light intensity at a given radial distance from the optic axis, light in the corresponding annular ring concentric with the optic axis was collected and guided to a single photodetector. This was achieved with optical fibers whose ends were distributed in concentric rings in the focal plane of the transform lens. Most of the fibers in each ring were connected at their other end to a single photodetector. This effectively integrates the light over an annular ring and makes use of most of the scattered light available. In contrast, a linearly scanned detector utilizes a relatively small portion of the scattered light.

The preliminary experimental study was done with seven concentric rings of fibers in a bundle of approximately 7-mm diameter. This fiber-optic

array was constructed by assembling the fibers in a bundle and fastening them at one end. This end was then cut perpendicular to the axis of the bundle. This flat end was positioned in the focal plane of a lens and illuminated with a focused light beam. The fibers were sorted at the other end according to their radial distance from the center fiber. Each group of fibers thus approximated an annular ring at the end of the bundle in the focal plane of the lens. Fibers for each of seven radial distances were sorted and connected to photodetectors.

Because of the circular symmetry of the optical-fiber array, it was proposed that the output of such an array would be independent of the angular orientation of the object in a coherent optical processing system. For many objects, the desired information is contained in the radial distribution of the light in the optical Fourier transform. An example of such an object is an optical transparency of the row patterns in plowed fields. The optical Fourier transform of a transparency containing uniformly spaced rows consists of a series of brightly illuminated spots. The row spacing, which is a useful indicator in crop identification, is related to the radial distance between the spots and is independent of the angular orientation. Therefore, an annular ring detector would acquire the useful information without requiring a particular orientation of the object. This would facilitate real-time processing of such objects.

The purpose of the work described in this report has been to design and fabricate an annular ring detector consisting of optical fibers. The maximum width of each concentric ring was to be small enough to permit the resolution of a Ronchi ruling transform with a dot spacing of 150 μm .

(0.0059 inch)*. There were to be a minimum of 100 concentric rings covering an area of at least 2.54 cm (1.00 inch) diameter.

The contract originally specified that each ring of fibers would be terminated in a photodiode and that the array be scanned in a sequence of 8 bit digital words. The engineering model fiber-optics detector was initially based on a configuration involving the use of relatively inexpensive electromechanical stepping switches for sequencing the output of the photodetectors. Subsequent to our initiation of the project, system studies performed at MSFC revealed that NASA's intended application of the fiber-optics detector called for a comparatively fast scan rate of light intensity measurements. This requirement precluded the use of the electromechanical stepping switches, but could be accommodated by use of a fast, but expensive, electronic multiplexing circuit. When the requirement was made known to us we initiated the design of the high-speed circuit to determine the fabrication cost. It was estimated that approximately \$12,000 was required to construct the multiplexing circuit.

By mutual agreement it was decided that NASA would implement the fabrication of the electronic circuit and would terminate the fibers with photodiodes. The axially symmetric array of optical-fibers and the computer programs used to test the matrix inversion technique were delivered to NASA in December, 1972. This report describes the techniques used to design and assemble the fiber-optic array in order to satisfy NASA's requirements.

*The English system of units was used for the principal measurements and calculations.

II. Fiber-Optics Bundle

The basic optical fibers were obtained from Poly-Optics, Inc., in Santa Ana, California. They consist of a plastic material rather than glass. The core has a nominal diameter of 69 μm (0.0027 inch). The cladding thickness is 3.8 μm (0.00015 inch) and the diameter of the resultant fiber is 76 μm (0.003 inch). The core accounts for 83% of the cross-sectional area and the cladding for 17%. The acceptance angle of the fibers is 70 degrees. The end losses are 10% for each end, and the line loss is 10% per foot.

The radius of the face of the fiber bundle is 1.34 cm (0.528 inch). However, only the fibers within a radius of 1.28 cm (0.504 inch) have been sorted into concentric circles. The useful section of the bundle consists of 168 adjacent rings concentric with the center fiber. Approximately 89,000 fibers make up the 168 rings. The rings are numbered so that ring number 1 is in contact with the center fiber, and ring number 168 is the outermost ring of fibers. The radius of the center of a ring is given by

$$R(\text{in.}) = 0.003 \times N$$

where N is the ring number. Therefore, the outermost (168th) ring has a radius of 1.28 cm (0.504 inch), or a diameter of 2.56 cm (1.008 inches). The length of the fibers (measured from the back of the collar) is 40 cm (15 3/4 inches).

The contract called for resolution of a Ronchi ruling transform that has a dot spacing of 150 μm . Sampling two or more times per interval in order to provide adequate resolution required that there be two or more

rings per 150 μm radial interval. This requirement was met by employing 76 μm (0.003 inch) diameter fibers and by making each annular ring only one fiber diameter wide. Therefore, the average width of each annular ring is 76 μm (0.003 inch).

As a test for several of the procedures to be used when fabricating the bundle of 76 μm (0.003 inch) diameter fibers, a similar bundle of 127 μm (0.005 inch) diameter fibers was assembled first. The fibers were sent by the manufacturer in 1.8 m (6 feet) long bundles of 6.4 mm (0.25 inch) diameter. Four such bundles of the 127 μm (0.005 inch) diameter fibers were each cut into four equal lengths of 46 cm (18 inches). The sixteen segments were then assembled as a single bundle and clamped at one end. The End Treatment Compound provided by the manufacturer was then applied to the clamped end of the bundle in order to bind all the fibers together. After this was done, the fibers in the bundle were combed until they were as parallel to each other as possible. This was done to optimize the light collection at the face of the completed bundle. The bundle of parallel fibers was then re-clamped, and the uneven end was cut off perpendicular to the axis. The new, even end of the bundle was then sanded flat and smooth.

When the collar for the final bundle of 76 μm (0.003 inch) diameter fibers was completed, this bundle was assembled following a procedure similar to that described above. After the compound had dried, the fibers in this bundle were combed straight and parallel, and then the machined collar was installed. Because the exact number of fibers and the packing efficiency attainable were not known, the bundle had been assembled with an

excess of fibers. The excess was carefully removed until the collar just fit and compressed the bundle as tightly as possible. The uneven end of the bundle was cut off about 6 mm (0.25 inch) from the face of the aluminum collar. The voids between the fibers were filled in by several applications of the End Treatment Compound. The face of the fiber-optic bundle was polished and the aluminum collar around the fibers was mounted in the optical component rotator. The rotator was mounted at the end of an optical bench in such a way that the fibers hung vertically. A 10X microscope objective was also mounted vertically. It was positioned just above the face of the fiber-optic bundle. The microscope objective was mounted on an X, Y, Z micropositioner which was mounted on a movable platform on the optical bench. A 90° prism was used to direct a horizontal helium-neon laser beam down through the microscope objective. The lens focused the light to a spot small enough to illuminate a single fiber at a time.

When the system was first set up, many fibers in the larger diameter circles were illuminated simultaneously. The reason for this was that the fibers had been compressed too tightly, distorting the core-jacket configuration to such an extent that the fibers were not light-pipes in the region of compression. The collar was removed, and the number of fibers in the bundle was reduced until the collar fit with just enough compression to hold the fibers without distorting them. The collar was re-mounted about 5 cm (2 inches) from its initial location on the bundle, and the short section where the fibers had been distorted was cut-off. The new face of the bundle was sanded and polished before the bundle was mounted on the optical bench. The optical quality of the fibers was found to be

uniformly good across the whole area of the face. The center fiber along the axis of rotation of the bundle was located and tagged. This fiber served as the basic reference for all radial measurements. Next, single fibers at each of three radial distances from the center fiber were located and tagged. These distances were 3.81 mm (0.150 inch), 7.62 mm (0.300 inch), and 11.43 mm (0.450 inch). These fibers served as convenient secondary reference points for measuring the radii of the circles.

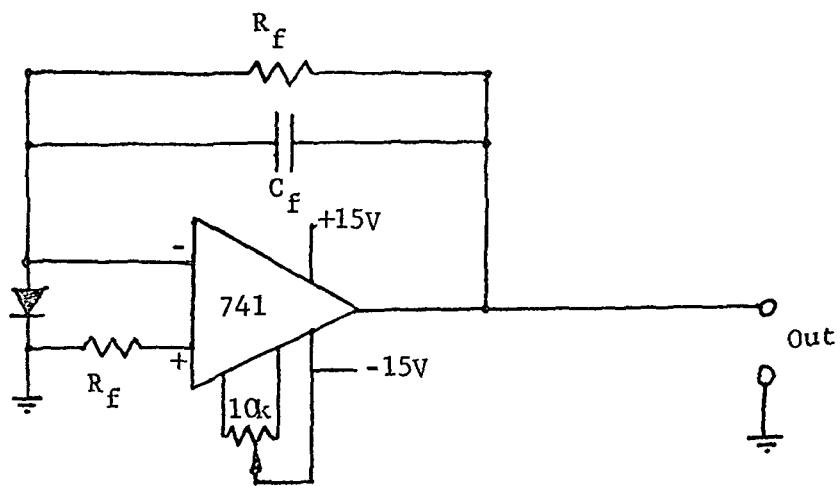
After the outermost six rings of fibers had been sorted, an analysis of the results indicated a potential problem. Because of the axial distance required to accumulate all the fibers associated with a particular ring into an individual bundle, the ultimate length required to sort all the rings would have been far too great. This would not have allowed us to separate the fibers in the innermost section of the bundle. The apparent reason for the problem was that the outermost fibers that had not been sorted into rings were becoming entangled in the bundle and making sorting difficult. In order to remedy the situation, all of the fibers outside the 2.56 cm (1.008 inches) diameter area of interest were cut just past the aluminum mounting collar and removed. With the extra care used in the sorting process, the axial length of the fiber bundle required for each ring was reduced to about 1 mm/ring.

III. Photodetectors

Data were recorded comparing the characteristics of a photodiode with those of a photoconductor. The photodiode tested was an EG&G model SGD-040B, and the photoconductor was a Clairex model CL 907HL CdS photocell. A helium-neon laser was used as a source of 0.6328 μm wavelength radiation, and calibrated neutral-density filters were used to attenuate the beam known amounts. The SGD-040B photodiode output was amplified by a Type 741-C operational amplifier, whose bandpass was designed to extend from dc to 40 Hz. All the measurements were made without chopping the light beam to eliminate the effects of the photoconductor time constant. The photodiode op-amp combination yielded an output voltage that was linear with respect to light intensity over the range from 10 to 70 dB attenuation of the 0.85 mW laser beam. Noise was a limiting factor at 70 dB attenuation. The resistance of the CdS photocell was measured directly with a Keithley Electrometer, and showed a linear variation with respect to light intensity over the range from 23 to 80 dB attenuation of the 0.85 mW laser beam.

A comparison of the results showed that the dynamic ranges of the photodiode-amplifier combination and photoconductor were comparable. Further refinement of the photodiode amplifier circuit should increase its dynamic range. However, the optimum conditions for using the photodiode require that the light source be chopped at a rate of at least 1 kHz in order to minimize 1/f noise.

The operational amplifier circuit recommended by Solid State Radiations, Inc., is diagrammed in Figure 1. The op-amp is an easily available,



$R_f: 0.1 - 10M\Omega$

$$V_{out} = i_D R_f$$

where i_D = detector current

10k potentiometer for setting zero output

C_f determines upper frequency response

$$3 \text{ dB point given by } f_c = \frac{1}{2\pi R_f C_f}$$

Figure 1.

relatively inexpensive Type 741-C. The other circuit components are a 10 k Ω potentiometer for zero-offset adjustment, two equal value resistors which determine the gain, and a capacitor which determines the high-frequency cutoff. The photodiode is operated in the unbiased or photovoltaic mode. Because the photodiode output is connected directly to the low impedance terminal of the op-amp circuit, the amplifier output voltage is directly proportional to the light intensity. Since a low impedance is required for high linearity of response, it is preferable to amplify the output of each photodiode before multiplexing. The available multiplexers can introduce about 100 ohms resistance. This would be more significant at the photodiode output than at the amplifier output.

The operational amplifier circuit was modified to increase the gain. Previously, 120 k Ω resistors were used in the feedback loop and between the non-inverting input and ground. These were replaced with 1 M Ω resistors in order to achieve a higher gain. However, a small problem was introduced along with the increased gain. The dc offset could no longer be adjusted for zero output with the detector in total darkness. This problem was remedied by reconnecting the original 120 k Ω resistor between the non-inverting input of the op-amp and ground. After this had been accomplished and found to be satisfactory, measurements were made of output voltage versus input light intensity. These measurements employed calibrated attenuators for controlling the light intensity over the range from 3 to 50 dB. The power of the laser beam available was 0.07 mW. The range of light intensity to which the photodiodes were exposed was 0.035 mW to 7×10^{-7} mW. The results are tabulated below for two different photodiodes.

<u>Laser Beam</u>		<u>Op-Amp Output Voltage</u>	
Attenuation (dB)	Power (μW)	EG&G SCD-040B (Volts)	PIN-30 (Volts)
0	70		
3	35	9.77	7.15
5	22	5.99	4.91
10	7.0	1.94	1.70
15	2.2	0.688	0.624
20	0.70	0.207	0.153
25	0.22	0.070	0.058
30	0.070	0.022	0.018
35	0.022	0.0076	0.0068
40	0.0070	0.0024	0.0023
50	0.0007	0.0003	0.0002

Measurement of low-level intensities requires that the photodiodes be operated in the unbiased photovoltaic mode, for which there is no dark current. Some of the most helpful descriptions of PIN photodiode characteristics are included in the following references [4,5,6,7].

IV. System Design

Several techniques for scanning the output from the center fiber and the 168 rings of fibers were considered. The original idea was to terminate each of the 168 rings with a photodetector and scan the array of detectors with stepping switches. However, the contact resistances might have been great enough to cause problems. An expensive solution would have been to use mercury wetted reed relays or an electronic multiplexing circuit. A less expensive and perhaps more versatile solution would consist of measuring the light from each ring of fibers with a single detector. This could be accomplished by arranging the groups of fibers from each ring in a circle and using a stepper motor to move the detector from one group of fibers to the next. Some advantages of this technique are that the detector could be changed easily in the event that different detector characteristics were desired, and the variation in dark current and sensitivity between detectors would be eliminated. This method, however, would be relatively slow in comparison with the electronic circuit described below.

Subsequent to our initiation of the project, system studies performed at MSFC revealed that NASA's intended application of the fiber-optics detector called for a relatively fast scan rate of light intensity measurements. This requirement could be met with the electronic multiplexing circuit described here. By mutual agreement, it was decided that NASA would implement the fabrication of the circuit.

A block diagram of a preliminary design for the entire system is shown in Figure 2. The block at the top represents the container for the bundle of optical fibers. In this drawing, the exposed face of the fiber bundle

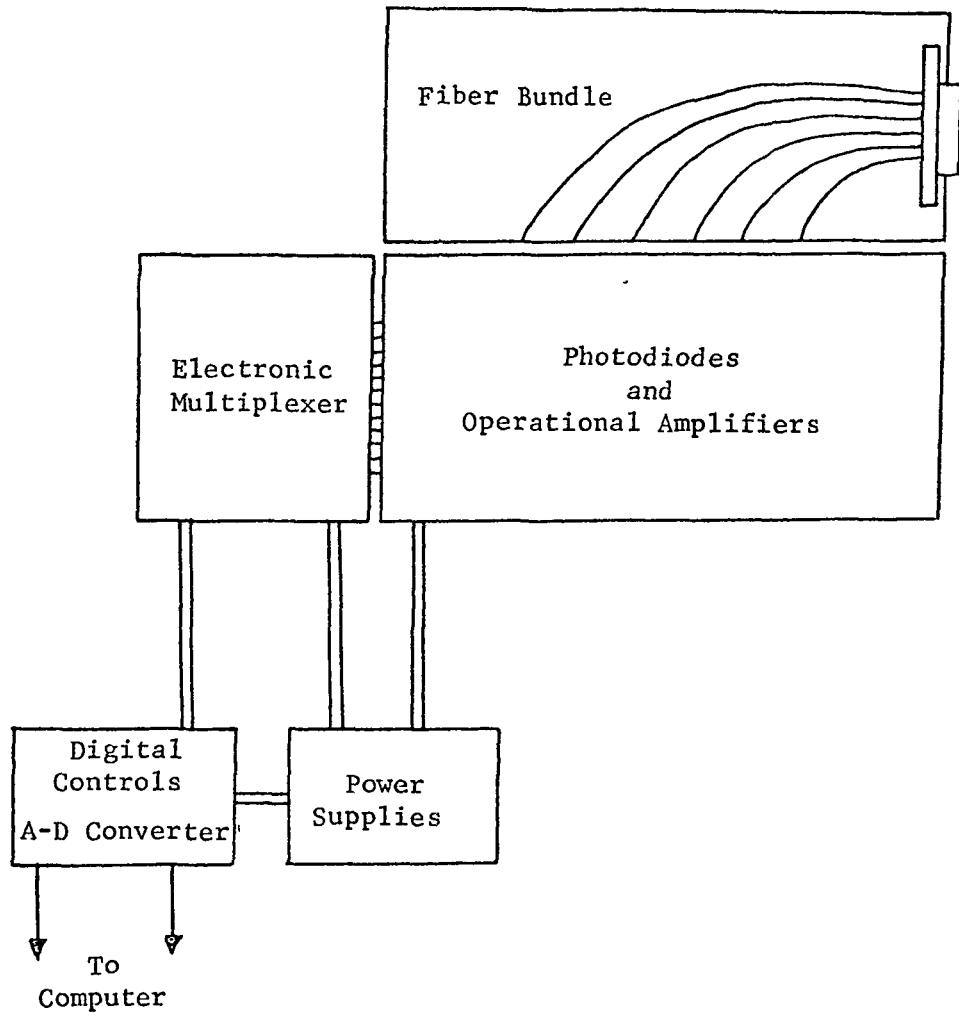


Figure 2 .

could be observed by visually scanning the array of fibers terminated in the wall of the container. If the direction of the incident light and the physical access to the system require it, the orientation of the bundle can be reversed.

Because of the requirement for scanning the detector output 30 times a second, it was suggested to NASA that photodiodes be used as the sensitive elements because of their fast response. Both the photoconductors and the electromechanical stepping switches discussed earlier are too slow for this scan rate. A faster, electronic multiplexing circuit has been designed. This circuit would be mounted to the photodiode/op-amp container as indicated in Figure 2 in order to minimize the cables required.

The coupling loss was measured between a fiber and a photodiode mounted according to our plans as compared with a fiber potted directly to the active area of the photodiode. These measurements were made for two different photodiodes: a United Detector Technology PIN-3D and an EG&G SGD-040B. The first measurements compared the coupling from the fiber to a detector whose window was not in contact with the end of the fiber with one that was. The PIN-3D showed a 4.2 dB improvement and the SGD-040B a 1.5 dB improvement. The difference may be due to the fact that the window of the SGD-040B was only 1.6 mm (1/16 inch) away from the fiber end and the PIN-3D was about 3 mm (1/8 inch) away. The photodiode windows were removed with a modified tubing cutter. The end of the fiber was then glued directly to the active area of each photodiode, one at a time. The PIN-3D showed a 3.6 dB improvement over the case where the fiber was in contact with the window, and a 7.8 dB improvement over the

case where there was a 3 mm (1/8 inch) air gap between the fiber and the window. The SGD-040B showed at least a 6 dB improvement over the case of direct contact with the window, and a 7.5 dB improvement over the measurement with a 1.6 mm (1/16 inch) air gap between the fiber and the window.

If NASA decides to take advantage of the improved coupling by potting the fibers directly to the active elements of the photodiodes, it will be necessary to obtain diodes with active areas that are compatible with the cross-sectional areas of the fiber bundles. It will also be necessary to modify the construction of the container for the optical fibers. With the photodiodes potted directly to the fiber bundles, a suitable means of connecting the diodes to the op-amp package will have to be devised.

It was suggested by NASA that the orientation of the container be horizontal rather than vertical. This will facilitate mounting the PC boards in the electronic package and should not present any unusual problems. It was also suggested that we keep all the fibers the same length in order to insure that the optical losses are the same. This might significantly increase the amount of cross-coupling between rings of fibers and be the cause of congestion within the container. However, neither of these possible problems is insurmountable.

We suggest that, if possible, the electronics be designed to measure variations in light levels over a 3 decade range. The full 7-10 decade linear dynamic range of the photodiodes could be utilized by varying the incident laser power in 30 dB steps.

A digital control circuit for the faster multiplexing circuit, and an A-D converter could be housed in a separate container and connected

to the other units by cables as indicated. The digital control circuit, multiplexer and A-D converter are shown in more detail in Figure 3. The multiplexer would consist of 11 separate 16-channel multiplexers for a total of 176 possible analog input channels. The fiber-optic system will consist of 168 concentric rings and the center fiber for a total of 169 data channels. The data channel selected by the multiplexer would be connected to a sample-and-hold circuit followed by an A-D converter. Because of the relatively slow response of logarithmic amplifiers, it is suggested that a 12 bit A-D converter without a log amp be used rather than a combination of a logarithmic amplifier and 8 bit converter. The 12-bit ADC would utilize more of the dynamic range of the photodiodes than would the 8-bit ADC.

The digital control circuit would require a synchronization signal to initiate an automatic scan for each change of the picture frame. The system could also be operated in a manual scan mode. In the manual mode any data channel could be selected at random by setting two thumbwheel switches. Also, the multiplexer could be manually stepped one channel at a time.

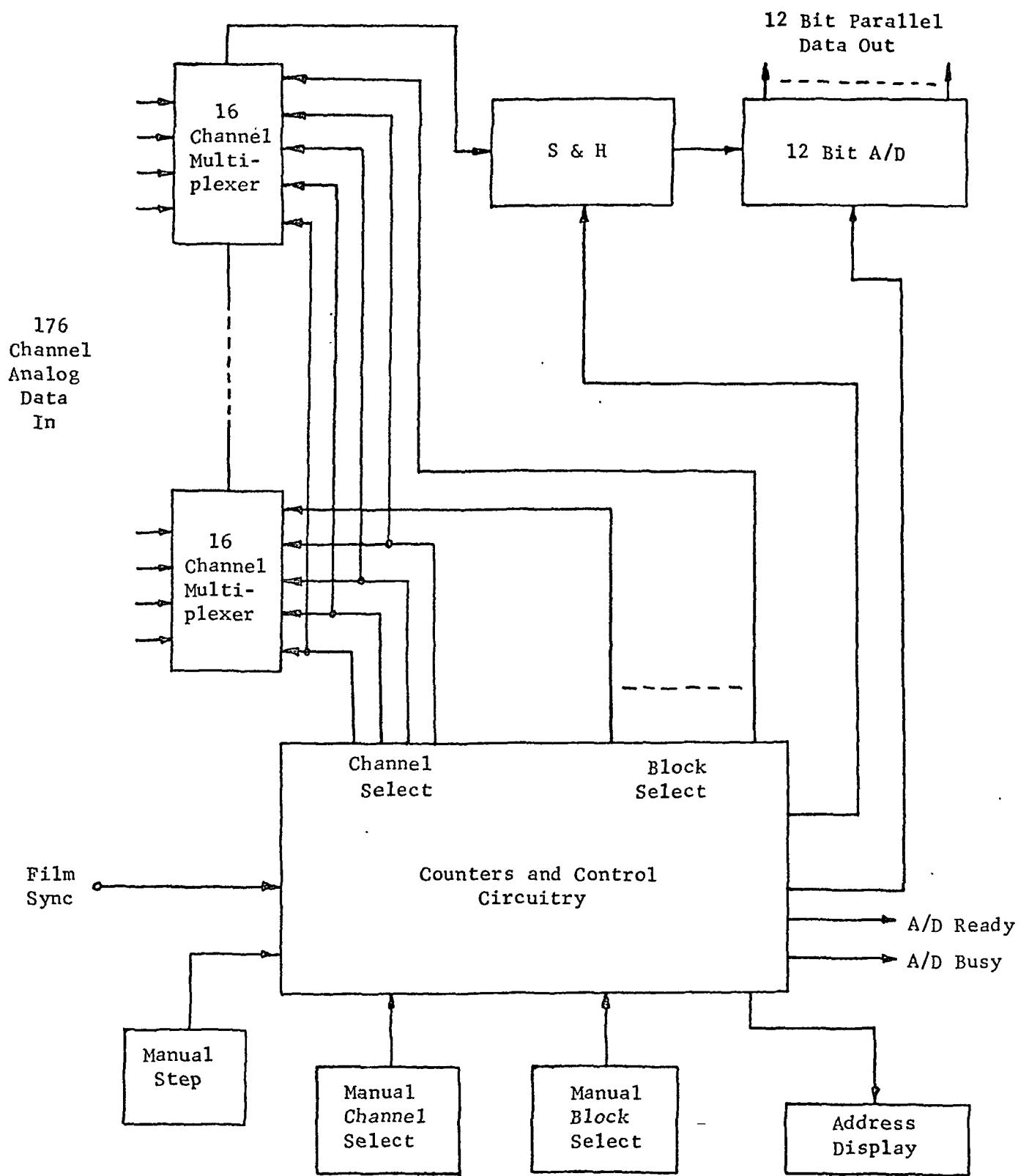


Figure 3.

V. Computer Programs

A preliminary program has been written and used to estimate the light flux measured by each ring of fibers in the far-field diffraction patterns produced by Ronchi rulings. Each tabulation of 168 values was used to calculate a column in the matrix $[G]$, where $[G][A] = [F]$ relates the areas, A_i , of known Ronchi rulings with the light flux values, F_j . Then the matrix $[G^T G]^{-1} [G]^T = [W]$ was computed where $[G^T]$ is the transpose of $[G]$. The matrix $[W]$, post-multiplied by a matrix $[F]$ of light flux measurements will yield a matrix $[A]$ whose elements are proportional to the areas of each Ronchi ruling used to generate a particular matrix $[F]$ [8]. That is, $[A] = [W][F] = [G^T G]^{-1} [G]^T [F]$. If $[G]$ were a square matrix, then $[G]^{-1} = [W]$. But, in general, $[G]$ will not be square, and the least square error solution for $[A] = [W][F]$ will be given by $[W] = [G^T G]^{-1} [G]^T [F]$ [8].

A Fortran program for calculating $[W]$ from $[G]$ was written and tested. The test consisted of calculating the light intensity expected at the 168 fiber rings for each of three different Ronchi rulings. The line spacings of the rulings were 80, 150, and 300 line pairs per inch. These three sets of data were used to derive an inversion matrix. Each of the three data sets yielded the correct results when multiplied by the inversion matrix. Also, the sum of the three initial data sets multiplied by the inversion matrix yielded the correct results. The final test was performed by adding noise to the sum of the three initial data sets. With a peak-to-peak noise equal to 1% of the largest light intensity, the largest error in the result was 2.25%. The amount of noise used was comparable with a usable dynamic range of only 2 decades.

VI. Conclusions and Recommendations

In the fiber-optic array fabricated on this contract, the fibers within a circular area of 2.56 cm (1.008 inches) diameter have been sorted into 168 adjacent rings concentric with the center fiber. Approximately 89,000 fibers of 76 μm (0.003 inch) diameter make up the 168 rings.

The configuration of this detector and the tedious work required for its fabrication make it a unique component which will facilitate measurements in optical processing systems and in light-scattering experiments.

It is recommended that analytical studies be undertaken to determine the ultimate performance and limitations of the annular ring detector in various optical processing schemes. Of particular importance is the mathematical operations which must be performed on the detector outputs to extract the desired information from the diffraction pattern falling on the face of the detector. The effects of noise, cross coupling, and unequal loss in the fiber bundles should be included to obtain realistic performance predictions.

Applications for the annular ring detector, other than its planned use in an optical processing system, should also be investigated. For example, such a detector is ideally suited for use in optical measurements of aerosol and hydrosol distributions. There is considerable interest in improving the measurement of the size distribution of sediment particles in rivers and streams and also of aerosol particles in the atmosphere. Some of the techniques presently in use require a separate knowledge of the optical properties of the materials of which the particles consist. Very often this information is not available in field measurements.

However, "the angular distribution of scattered intensity within the center portion of the forward lobe is a useful indication of the diameter of particles whose refractive index is not known" [9]. It was this particular application that led to our development of a circularly symmetric fiber-optic detector array. This development was necessitated by the fact that there is no commercially available annular ring detector with comparable resolution.

The author, under an in-house program, is presently considering several different techniques for analyzing the output of annular ring detectors to yield a measure of particle size. The goal of this internally funded work is to calculate the size- and number-resolutions of just the matrix inversion technique. However, if additional funding could be obtained, a comparison of all the techniques would be made for each of several experimental conditions. This would facilitate selection of the optimum technique for a given experimental situation. The ultimate goal of this work is to develop field instrumentation capable of continuously measuring the size distribution of particles in air or water. Such work could constitute an example of the utilization of technology developed with NASA support.

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